

Constraint on $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances from their mixing intensity

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Structure of the $a_0(980)$ and $f_0(980)$ resonances is investigated with the $a_0(980)$ - $f_0(980)$ mixing intensity from the viewpoint of compositeness, which corresponds to the amount of two-body states composing resonances as well as bound states. For this purpose we first formulate the $a_0(980)$ - $f_0(980)$ mixing intensity as the ratio of two partial decay widths of a parent particle, in the same manner as the recent analysis in BES experiments. Calculating the $a_0(980)$ - $f_0(980)$ mixing intensity with the existing Flatte parameters from experiments, we find that many combinations of the $a_0(980)$ and $f_0(980)$ Flatte parameters can reproduce the experimental value of the $a_0(980)$ - $f_0(980)$ mixing intensity by BES. Next, from the same Flatte parameters we also calculate the $K\bar{K}$ compositeness for $a_0(980)$ and $f_0(980)$. Although the compositeness with the correct normalization becomes complex in general for resonance states, we find that the Flatte parameters for $f_0(980)$ imply large absolute value of the $K\bar{K}$ compositeness and the parameters for $a_0(980)$ lead to small but nonnegligible absolute value of the $K\bar{K}$ compositeness. Then, connecting the mixing intensity and the $K\bar{K}$ compositeness via the $a_0(980)$ - and $f_0(980)$ - $K\bar{K}$ coupling constants, we establish a relation between them. As a result, a small mixing intensity indicates a small value of the product of the $K\bar{K}$ compositeness for the $a_0(980)$ and $f_0(980)$ resonances. Moreover, the experimental value of the $a_0(980)$ - $f_0(980)$ mixing intensity implies that the $a_0(980)$ and $f_0(980)$ resonances cannot be simultaneously $K\bar{K}$ molecular states.

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I. INTRODUCTION

The nature of the lightest scalar meson nonet [$f_0(500)$ or σ , $K_0^*(800)$ or κ , $f_0(980)$, and $a_0(980)$] has been a hot topic in hadron physics for many years [1]. A naïve expectation with the $q\bar{q}$ configuration indicates that they should show the same mass ordering as, *e.g.*, the vector meson nonet, but in real they exhibit inverted spectrum from the expectation. For this reason they have been considered to be exotic hadrons, which are not able to be classified as $q\bar{q}$ for mesons and qqq for baryons. Indeed, in Refs. [2, 3] it was suggested that in a bag model the interaction between quarks inside a compact $qq\bar{q}\bar{q}$ system is attractive especially in the scalar channel and hence the light scalar mesons would be compact $qq\bar{q}\bar{q}$ systems. However, it was found that in a non-relativistic quark model $K\bar{K}$ molecules can appear as weakly bound *s*-wave states, which may be identified with $f_0(980)$ and $a_0(980)$ [4, 5]. The lightest scalar mesons can also be described by the combination of the chiral perturbation theory and the scattering unitarity [6–15] in pseudoscalar meson-pseudoscalar meson scatterings from the hadronic degrees of freedom. This fact implies that

the lightest scalar mesons may have nonnegligible components of hadronic molecules. In a model-independent way, on the other hand, the structure of $f_0(980)$ and $a_0(980)$ was discussed in Ref. [16], which suggested that $f_0(980)$ should be a $K\bar{K}$ molecular state to a large degree and $a_0(980)$ also seems to have a nonnegligible $K\bar{K}$ component. There are further discussions on their structure as well, *e.g.*, hybrid states for $f_0(980)$ and $a_0(980)$ [17].

Among the light scalar mesons, $a_0(980)$ and $f_0(980)$ are of special interest because their almost degenerate masses would lead to a mixing of these mesons in isospin symmetry violating processes. In particular, it was pointed out in Ref. [18] that the difference of the unitarity cuts between the charged and neutral $K\bar{K}$ pairs, which thresholds are close to the $a_0(980)$ and $f_0(980)$ masses, can enhance the $a_0(980)$ - $f_0(980)$ mixing to be sizable compared to, *e.g.*, the $\rho(770)$ - $\omega(782)$ mixing. Namely, the leading contribution to the $a_0(980)$ - $f_0(980)$ mixing comes from the mixing amplitude $\Lambda_1 + \Lambda_2$ in Fig. 1, and it behaves

$$\Lambda_1 + \Lambda_2 = \mathcal{O}(p_{K^0} - p_{K^+}), \quad (1)$$

where p_{K^0} (p_{K^+}) denotes the magnitude of the relative momentum of the neutral (charged) kaon pair. Then, due to the difference of the unitary cuts, the mixing effect should be unusually enhanced in the energy between $m_{K^+} + m_{K^-} = 987$ MeV and $m_{K^0} + m_{\bar{K}^0} = 995$ MeV to

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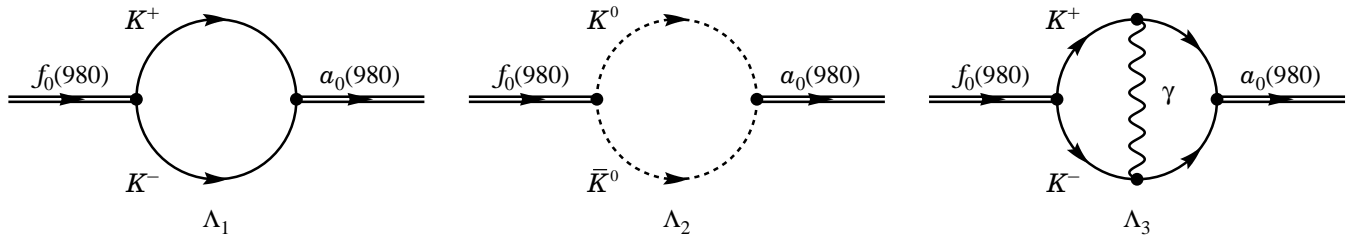


FIG. 1: Feynman diagrams for the leading contribution ($\Lambda_1 + \Lambda_2$) and a subleading contribution (Λ_3) to the $a_0(980)$ - $f_0(980)$ mixing.

be

$$\Lambda_1 + \Lambda_2 = \mathcal{O} \left(\sqrt{\frac{m_{K^0}^2 - m_{K^+}^2}{m_{K^0}^2 + m_{K^+}^2}} \right), \quad (2)$$

while out of the energy region the mixing effect returns to a value of natural size, $\mathcal{O}[(m_{K^0}^2 - m_{K^+}^2)/(m_{K^0}^2 + m_{K^+}^2)]$. In addition, as a subleading contribution the electromagnetic interaction would enhance the $a_0(980)$ - $f_0(980)$ mixing, since the electromagnetic interaction takes place selectively in the K^+K^- loop. Bearing in mind that in general a scalar meson does not have derivative couplings to two pseudoscalar mesons, we have only a soft photon exchange between K^+ and K^- as the leading order with respect to the electromagnetic interaction, which is diagrammatically shown as Λ_3 in Fig. 1. Indeed, the amplitude Λ_3 logarithmically diverges at the K^+K^- threshold in an approximation of the threshold expansion. For observations of the $a_0(980)$ - $f_0(980)$ mixing, various reactions which should be sensitive to the mixing were discussed in, *e.g.*, Refs. [19–27], and the mixing effect was recently observed in an experiment [28] from the decay of J/ψ .

The $a_0(980)$ - $f_0(980)$ mixing has been expected to shed light on the structure of the $a_0(980)$ and $f_0(980)$ resonances. Actually in Ref. [28] the experimental value of the mixing intensity was compared to several theoretical predictions and structure of the two resonances was discussed. We here emphasize that coupling constants of $a_0(980)$ - $K\bar{K}$ and $f_0(980)$ - $K\bar{K}$ reflect the $K\bar{K}$ structure of the $a_0(980)$ and $f_0(980)$ resonances, respectively; especially a larger $K\bar{K}$ coupling constant means a larger fraction of the $K\bar{K}$ component in the scalar mesons [29]. In recent studies this statement has been formulated in terms of compositeness [30–36] in the so-called chiral unitary approach, which is a way to combine the chiral perturbation theory and the scattering unitarity. In these studies the compositeness was defined as the two-body composite part of the normalization of the total wave function, and hence the compositeness corresponds to the amount of the two-body states composing a resonance as well as a bound state. In the formulation the two-body wave function was found to be proportional to the coupling constant of the resonance state to the two-body state [36–38]. Thus, bearing in mind that the $a_0(980)$ - $f_0(980)$ mixing amplitude contains both the cou-

pling constants of $a_0(980)$ - $K\bar{K}$ and $f_0(980)$ - $K\bar{K}$, one can expect a relation between the $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances and their mixing intensity through the strength of the coupling constants of $a_0(980)$ - $K\bar{K}$ and $f_0(980)$ - $K\bar{K}$ in the mixing amplitude, in a similar manner to the relation between the $\Lambda(1405)$ radiative decay width and its $\bar{K}N$ compositeness established in Ref. [39]. The purpose of this paper is to establish a relation between the $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances and their mixing intensity and to give a constraint on the structure of the two resonances from the experimental value of the mixing intensity obtained in Ref. [28].

This paper is organized as follows. In Sec. II we formulate the $a_0(980)$ - $f_0(980)$ mixing intensity. In this section we also calculate the $a_0(980)$ - $f_0(980)$ mixing intensity with several Flatte parameter sets for $a_0(980)$ and $f_0(980)$ from experiments, and compare the numerical results with the recent experimental result. Next, in Sec. III we develop our formulation of the compositeness in the context of the chiral unitary approach, and we calculate the $K\bar{K}$ compositeness of $a_0(980)$ and $f_0(980)$ with the experimental Flatte parameter sets. Then, in Sec. IV we give a relation between the mixing intensity and the $K\bar{K}$ compositeness for the $a_0(980)$ and $f_0(980)$ resonances. Moreover, we discuss further steps for the determination of the structure of the $a_0(980)$ and $f_0(980)$ resonances in Sec. V. Section VI is devoted to the conclusion of this study.

II. THE $a_0(980)$ - $f_0(980)$ MIXING INTENSITY

In this section we formulate the $a_0(980)$ - $f_0(980)$ mixing intensity. For this purpose we first determine the expression of the $a_0(980) \leftrightarrow f_0(980)$ mixing amplitude in Sec. II A. Next we evaluate the propagators of $a_0(980)$ and $f_0(980)$ with their mixing in Sec. II B. Then we formulate the $a_0(980)$ - $f_0(980)$ mixing intensity as the ratio of partial decay widths of a parent particle in Sec. II C. Finally in Sec. II D we calculate the mixing intensity by using several parameter sets obtained from experimental data.

A. Mixing amplitude

First of all we determine the $a_0(980) \leftrightarrow f_0(980)$ mixing amplitude $\Lambda(s)$ as a function of the squared momentum of the scalar mesons, s . In this study we consider three Feynman diagrams in Fig. 1 for the $a_0(980) \leftrightarrow f_0(980)$ mixing, and the mixing amplitude $\Lambda(s)$ is sum of the three contributions:

$$\Lambda(s) = \Lambda_1(s) + \Lambda_2(s) + \Lambda_3(s). \quad (3)$$

Here we assume isospin symmetry for coupling constants. Namely, $a_0(980)$ - $K\bar{K}$ and $f_0(980)$ - $K\bar{K}$ coupling constants in the particle basis, \bar{g}_a and \bar{g}_f , are given as¹

$$\bar{g}_a = \bar{g}_{aK^+K^-} = -\bar{g}_{aK^0\bar{K}^0}, \quad \bar{g}_f = \bar{g}_{fK^+K^-} = \bar{g}_{fK^0\bar{K}^0}. \quad (4)$$

Then it was pointed out in Ref. [18] that the sum of the first and second contributions, $\Lambda_1 + \Lambda_2$, converges and the result can be presented as an expansion in the $K\bar{K}$ phase space:

$$\begin{aligned} \Lambda_1(s) + \Lambda_2(s) = & -\frac{i}{16\pi} \bar{g}_a \bar{g}_f [\sigma_1(s) - \sigma_2(s)] \\ & + \mathcal{O}[\sigma_1^2(s) - \sigma_2^2(s)], \end{aligned} \quad (5)$$

where $i = 1$ (2) denotes the channel K^+K^- ($K^0\bar{K}^0$) and the phase space $\sigma_i(s)$ is defined as

$$\sigma_i(s) \equiv \frac{\lambda^{1/2}(s, m_i^2, m_i^2)}{s} = \sqrt{1 - \frac{4m_i^2}{s}}, \quad i = 1, 2, \quad (6)$$

with the Källen function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ and masses $m_1 = m_{K^+}$ and $m_2 = m_{K^0}$.² Since we have taken into account just the difference of the unitary cut contributions, this leading-order contribution is model independent except for the coupling constants.

The third contribution to the mixing amplitude, Λ_3 , is a soft photon-exchange diagram between K^+K^- , and with the photon-exchange loop function $G_\gamma(s)$ the mixing amplitude can be written as

$$\Lambda_3(s) = \bar{g}_a G_\gamma(s) \bar{g}_f. \quad (7)$$

For the evaluation of the photon-exchange loop function $G_\gamma(s)$, we take an approximation by the threshold-expanded form [40], which reads [24]

$$\begin{aligned} G_\gamma(s) = & -\frac{\alpha}{32\pi} \left[\ln \frac{4m_{K^+}^2 - s}{m_{K^+}^2} + \ln 2 + \frac{21\zeta(3)}{2\pi^2} \right] \\ & + \mathcal{O}[(s - 4m_{K^+}^2)^2], \end{aligned} \quad (8)$$

¹ We put bar on the coupling constants, $\bar{g}_{a,f}$, which are used on the real energy axis. On the other hand, we will not put bar on the coupling constants which are evaluated as the residue of the scattering amplitude (see Sec. III).

² In our calculations we use the physical masses $m_{K^+} = m_{K^-} = 493.68$ MeV, $m_{K^0} = m_{\bar{K}^0} = 497.61$ MeV, and $m_\eta = 547.85$ MeV, while for the isospin symmetric masses we use $m_\pi = (m_{\pi^+} + m_{\pi^-} + m_{\pi^0})/3 = 138.04$ MeV and $m_K = (m_{K^+} + m_{K^-} + m_{K^0} + m_{\bar{K}^0})/4 = 495.65$ MeV.

with the fine structure constant $\alpha \approx 1/137$ and the zeta function $\zeta(x)$ with $\zeta(3) = 1.20205\dots$

In above expressions, only the two coupling constants, \bar{g}_a and \bar{g}_f , are the parameters and reflect the structure of the $a_0(980)$ and $f_0(980)$ resonances. In this study the coupling constants are taken from the Flatte parameter sets with several experimental fittings in Sec. IID, and then in Sec. IV they are used to establish a relation between the mixing intensity and the $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances.

B. Propagators of $a_0(980)$ and $f_0(980)$ with their mixing

Next we formulate propagators of the $a_0(980)$ and $f_0(980)$ mesons with their mixing. If the $a_0(980)$ - $f_0(980)$ mixing is absent, their propagators can be expressed as $1/D_a(s)$ and $1/D_f(s)$ in the Flatte parametrization [41]

$$\begin{aligned} D_a(s) & \equiv s - M_a^2 + i\sqrt{s}[\Gamma_{\pi\eta}^a(s) + \Gamma_{K\bar{K}}^a(s)], \\ D_f(s) & \equiv s - M_f^2 + i\sqrt{s}[\Gamma_{\pi\pi}^f(s) + \Gamma_{K\bar{K}}^f(s)], \end{aligned} \quad (9)$$

for $a_0(980)$ and $f_0(980)$, respectively. Here s is the squared momentum of the scalar mesons, M_a and M_f are masses of $a_0(980)$ and $f_0(980)$, respectively, and decay width of $a \rightarrow b + c$, $\Gamma_{bc}^a(s)$, is defined as

$$\Gamma_{bc}^a(s) \equiv \frac{|\bar{g}_{abc}|^2}{8\pi s} p_{bc}(s), \quad p_{bc}(s) \equiv \frac{\lambda^{1/2}(s, m_b^2, m_c^2)}{2\sqrt{s}}, \quad (10)$$

with the a - bc coupling constant in the isospin basis \bar{g}_{abc} , the magnitude of the relative momentum p_{bc} , and the b and c masses m_b and m_c , respectively. Here we note that, due to the energy dependence of the decay-width terms in $D_{a(f)}$, the pole position of the propagator slightly shifts from that of the naïve expectation $s = [M_{a(f)} - i\Gamma_{bc}^{a(f)}(M_{a(f)}^2)/2]^2$. Furthermore, the momentum $p_{bc}(s)$ in the decay width (10) requires us to move to the proper Riemann sheet when we search for the pole of the propagator. Throughout this study we search for the $a_0(980)$ [$f_0(980)$] pole existing in the unphysical Riemann sheet of the $\pi\eta$ ($\pi\pi$) channel and in the physical Riemann sheet of the $K\bar{K}$ channel. We also note that the isospin symmetry breaking negligibly affects the decay width $\Gamma_{bc}^a(s)$ in this study, so we use the isospin symmetric masses and coupling constants of pions and kaons for the evaluation of the decay width (10).

Now let us turn on the $a_0(980)$ - $f_0(980)$ mixing. In this condition we can obtain the $a_0(980)$ propagator with the $a_0(980)$ - $f_0(980)$ mixing, $P_a(s)$, by summing up all the contributions of $a_0(980) \rightarrow f_0(980) \rightarrow \dots \rightarrow a_0(980)$, and hence $P_a(s)$ is expressed as

$$\begin{aligned} P_a(s) & = \frac{1}{D_a} + \frac{1}{D_a} \Lambda \frac{1}{D_f} \Lambda \frac{1}{D_a} + \dots = \frac{1}{D_a} \sum_{n=0}^{\infty} \left(\frac{\Lambda^2}{D_a D_f} \right)^n \\ & = \frac{1}{D_a} \left(1 - \frac{\Lambda^2}{D_a D_f} \right)^{-1} = \frac{D_f}{D_a D_f - \Lambda^2}, \end{aligned} \quad (11)$$

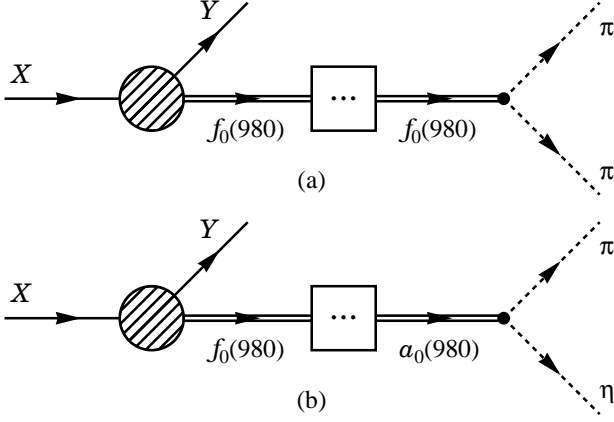


FIG. 2: Schematic diagrams of the decays (a) $X \rightarrow Y f_0(980) \rightarrow Y \pi \pi$ and (b) $X \rightarrow Y f_0(980) \rightarrow Y a_0(980) \rightarrow Y \pi \eta$. In the figures ellipses denote that the propagators of the scalar mesons include the $a_0(980)$ - $f_0(980)$ mixing contribution.

where $\Lambda(s)$ is the $a_0(980) \leftrightarrow f_0(980)$ mixing amplitude determined in the previous subsection. In similar manners we can obtain the $f_0(980)$ propagator $P_f(s)$, the $a_0(980) \rightarrow f_0(980)$ propagator $P_{a \rightarrow f}(s)$, and the $f_0(980) \rightarrow a_0(980)$ propagator $P_{f \rightarrow a}(s)$ with the $a_0(980)$ - $f_0(980)$ mixing, and they are summarized as follows:

$$\begin{pmatrix} P_a & P_{a \rightarrow f} \\ P_{f \rightarrow a} & P_f \end{pmatrix} = \frac{1}{D_a D_f - \Lambda^2} \begin{pmatrix} D_f & \Lambda \\ \Lambda & D_a \end{pmatrix}. \quad (12)$$

C. Partial decay widths and mixing intensity

Let us now define the $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} . In the experimental analysis in Ref. [28] the mixing intensity was defined as the ratio of two branching fractions of J/ψ , $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0(980) \rightarrow \phi \pi \eta$ to $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi \pi \pi$. Hence, in the same manner as this experimental analysis, we define the $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} as the ratio of the decay widths:

$$\xi_{fa} \equiv \frac{\Gamma_{X,a}}{\Gamma_{X,f}}, \quad (13)$$

where $\Gamma_{X,a}$ and $\Gamma_{X,f}$ are partial decay widths of a meson X to $Y f_0(980) \rightarrow Y a_0(980) \rightarrow Y \pi \eta$ and to $Y f_0(980) \rightarrow Y \pi \pi$, respectively. Here we assume that both X and Y are $I = 0$ states, and hence isospin symmetry allows only the X - Y - $f_0(980)$ vertex. Therefore, in our formulation $a_0(980)$ appears only through the $a_0(980)$ - $f_0(980)$ mixing. We could consider an intrinsic isospin-violating contribution which allows a direct coupling of X to the Y - $a_0(980)$ system, but such a contribution will scale as a natural size, for instance, $(m_d - m_u)/m_s$, and will be much smaller than the mixing amplitude of the $K\bar{K}$ loops between the charged and neutral $K\bar{K}$ thresholds. For this reason we neglect the direct X - Y - $a_0(980)$ cou-

pling.³ Schematic diagrams of the X decays to $Y \pi \pi$ and $Y \pi \eta$ are shown in Fig. 2. In this study we do not take into account final-state interactions between Y and pseudoscalar mesons by assuming that decay points of $f_0(980)$ and $a_0(980)$ are well isolated from the particle Y . As we will see, the final expression of the mixing intensity ξ_{fa} does not contain masses nor widths of the particles X and Y .

Since the decay process $X \rightarrow Y f_0(980) \rightarrow Y \pi \pi$ has three-body final state, the width $\Gamma_{X,f}$ can be calculated as [1]

$$\Gamma_{X,f} = \frac{1}{(2\pi)^5 16 M_X^2} \int dM_{\pi\pi} p_{\text{cm}}(M_{\pi\pi}) p_Y(M_{\pi\pi}) \times \int d\Omega \int d\Omega_Y |T_f|^2, \quad (14)$$

where M_X is the mass of the particle X , $M_{\pi\pi}$ is the invariant mass of the $\pi\pi$ system in the final state, and Ω and Ω_Y are solid angles for the final π in the $\pi\pi$ rest frame and for the final Y in the X rest frame, respectively. Momenta of final-state π in the $\pi\pi$ rest frame, p_{cm} , and Y in the X rest frame, p_Y , are defined as,

$$p_{\text{cm}}(M) = \frac{\lambda^{1/2}(M^2, m_\pi^2, m_\pi^2)}{2M}, \quad (15)$$

$$p_Y(M) = \frac{\lambda^{1/2}(M_X^2, M_Y^2, M^2)}{2M_X}, \quad (16)$$

respectively, with the particle Y mass M_Y . The decay amplitude T_f is expressed as

$$T_f = T_{\text{prod}}(M_{\pi\pi}) P_f(M_{\pi\pi}^2) \bar{g}_{f\pi\pi}, \quad (17)$$

where T_{prod} is the $f_0(980)$ production amplitude for the $X \rightarrow Y f_0(980)$ process, P_f is the $f_0(980)$ propagator with the $a_0(980)$ - $f_0(980)$ mixing given in Eq. (12), and $\bar{g}_{f\pi\pi}$ is the $f_0(980)$ - $\pi\pi$ coupling constant in the isospin basis. Then we assume that the $f_0(980)$ decay width Γ^f is small compared to the energy scales in which the momentum p_Y and the $f_0(980)$ production amplitude T_{prod} largely change. In this condition, since the $M_{\pi\pi}$

³ However, in certain cases we cannot neglect the direct X - Y - $a_0(980)$ coupling. Actually it is claimed in Refs. [26, 42] that the mixing intensity ξ_{fa} is affected by interferences between several diagrams of $a_0(980)$ and $f_0(980)$ productions and hence ξ_{fa} depends on the reaction, as experimentally observed in the $\eta(1405) \rightarrow f_0(980)\pi^0$ decay [43]. Nevertheless, in this study we employ the mixing intensity ξ_{fa} in Eq. (13) since such interferences are expected to enhance or decline both the $a_0(980)$ and $f_0(980)$ productions similarly and the mixing intensity ξ_{fa} will not change so much as long as only intrinsic isospin-violating contributions are considered. On the other hand, if the decaying particle exists close to thresholds such as $\bar{K}K^*$, these thresholds could be another source of isospin violation and provide a nonnegligible X - Y - $a_0(980)$ coupling.

integral is dominated by the $f_0(980)$ mass region due to the $f_0(980)$ propagator, we can approximate the Y momentum $p_Y(M_{\pi\pi})$ and the $f_0(980)$ production amplitude $T_{\text{prod}}(M_{\pi\pi})$ as the values at $M_{\pi\pi} = M_f$, respectively. Therefore, only the squared $f_0(980)$ propagator $|P_f(M_{\pi\pi}^2)|^2$ and the momentum $p_{\text{cm}}(M_{\pi\pi})$ appear in the $M_{\pi\pi}$ integral in the expression of the decay width (14):

$$\Gamma_{X,f} = \frac{1}{(2\pi)^5 16M_X^2} p_Y(M_f) \int d\Omega_Y |T_{\text{prod}}(M_f)|^2 \times \int dM_{\pi\pi} 4\pi p_{\text{cm}}(M_{\pi\pi}) |\bar{g}_{f\pi\pi}|^2 |P_f(M_{\pi\pi}^2)|^2, \quad (18)$$

where we have performed the integral of the solid angle Ω . Then by using the relation in Eq. (10) and the identity $p_{\text{cm}}(M) = p_{\pi\pi}(M^2)$, one can obtain

$$\Gamma_{X,f} = C \times \int dM_{\pi\pi} M_{\pi\pi}^2 \Gamma_{\pi\pi}^f(M_{\pi\pi}^2) |P_f(M_{\pi\pi}^2)|^2, \quad (19)$$

with a constant prefactor C :

$$C \equiv \frac{p_Y(M_f)}{16\pi^3 M_X^2} \int d\Omega_Y |T_{\text{prod}}(M_f)|^2. \quad (20)$$

In a similar manner, the width of the decay process $X \rightarrow Y f_0(980) \rightarrow Y a_0(980) \rightarrow Y \pi\eta$, $\Gamma_{X,a}$, can be calculated from

$$\Gamma_{X,a} = \frac{1}{(2\pi)^5 16M_X^2} \int dM_{\pi\eta} p'_{\text{cm}}(M_{\pi\eta}) p_Y(M_{\pi\eta}) \times \int d\Omega' \int d\Omega_Y |T_a|^2, \quad (21)$$

where p'_{cm} is the final-state π momentum in the $\pi\eta$ rest frame,

$$p'_{\text{cm}}(M) = \frac{\lambda^{1/2}(M^2, m_\pi^2, m_\eta^2)}{2M}, \quad (22)$$

Ω' is the solid angle for the final π in the $\pi\eta$ rest frame, and T_a is the decay amplitude evaluated as

$$T_a = T_{\text{prod}}(M_{\pi\eta}) P_{f \rightarrow a}(M_{\pi\eta}^2) \bar{g}_{a\pi\eta}. \quad (23)$$

Here $P_{f \rightarrow a}$ is the $f_0(980) \rightarrow a_0(980)$ mixing propagator given in Eq. (12) and $\bar{g}_{a\pi\eta}$ is the $a_0(980)$ - $\pi\eta$ coupling constant. Then, in order to evaluate the decay width $\Gamma_{X,a}$, we use the fact that the $a_0(980)$ - $f_0(980)$ mixing takes place particularly at the $\pi\eta$ invariant mass $M_{\pi\eta} \approx M_f \approx M_a \approx 2m_K$. This is because the $f_0(980) \rightarrow a_0(980)$ transition is dominated by the difference of the unitarity cuts for the charged and neutral $K\bar{K}$ thresholds and hence the mixing amplitude $\Lambda(M_{\pi\eta}^2)$ shows a narrow peak at the $K\bar{K}$ thresholds with a width $\sim (m_{K^0} + m_{\bar{K}^0}) - (m_{K^+} + m_{K^-}) \approx 8$ MeV. Therefore, one can take the values $M_{\pi\eta} \approx M_f \approx M_a$ for the amplitude T_{prod} and the momentum p_Y in $\Gamma_{X,a}$. Moreover, by using the relation in Eq. (10) we can replace the momentum $p'_{\text{cm}}(M_{\pi\eta})$ and the squared coupling constant $|\bar{g}_{a\pi\eta}|^2$

with the decay width $\Gamma_{\pi\eta}^a(M_{\pi\eta}^2)$ and a kinetic term, which results in

$$\Gamma_{X,a} = C \times \int dM_{\pi\eta} M_{\pi\eta}^2 \Gamma_{\pi\eta}^a(M_{\pi\eta}^2) |P_{f \rightarrow a}(M_{\pi\eta}^2)|^2, \quad (24)$$

where the constant C is same as that in Eq. (20).

As a consequence, we obtain the final expression of the $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} (13) as

$$\xi_{fa} = \frac{\int dM_{\pi\eta} M_{\pi\eta}^2 \Gamma_{\pi\eta}^a(M_{\pi\eta}^2) |P_{f \rightarrow a}(M_{\pi\eta}^2)|^2}{\int dM_{\pi\pi} M_{\pi\pi}^2 \Gamma_{\pi\pi}^f(M_{\pi\pi}^2) |P_f(M_{\pi\pi}^2)|^2}. \quad (25)$$

The range of the $M_{\pi\eta}$ integral is fixed so as to cover the $K\bar{K}$ thresholds, say $[0.96 \text{ GeV}, 1.02 \text{ GeV}]$. On the other hand, we fix the integral range of $M_{\pi\pi}$ so as to take into account the bump structure coming from the squared propagator $|P_f(M_{\pi\pi}^2)|^2$. In this formulation, the model parameters for the mixing intensity are the masses M_a and M_f in the propagators and the coupling constants \bar{g}_a , \bar{g}_f , $\bar{g}_{a\pi\eta}$, and $\bar{g}_{f\pi\pi}$. We note that the final expression of the mixing intensity ξ_{fa} does not contain masses nor widths of the particles X and Y , as one can expect that the $a_0(980)$ - $f_0(980)$ mixing intensity does not depend on the $f_0(980)$ production process.

Finally we mention that one can reproduce the mixing intensity given in Ref. [25] by considering only the integrands in Eqs. (25) and taking $M_{\pi\pi}^2 = M_{\pi\eta}^2 = s$, which results in

$$\xi_{fa}^{\text{WZ}}(s) = \frac{\Gamma_{\pi\eta}^a(s) |P_{f \rightarrow a}(s)|^2}{\Gamma_{\pi\pi}^f(s) |P_f(s)|^2}. \quad (26)$$

Actually, in Ref. [25] the authors calculated the mixing intensity ξ_{fa}^{WZ} at the central value of the two $K\bar{K}$ thresholds, $\xi_{fa}^{\text{WZ}}((m_{K^+} + m_{K^0})^2)$. Here we emphasize that the mixing intensity ξ_{fa}^{WZ} at the central value of the two $K\bar{K}$ thresholds would be larger than the numerical result from Eq. (25) with the same parameter set. This behavior comes from the fact that in Eq. (26) we do not perform the integral of $M_{\pi\pi}$ for the decay $f_0(980) \rightarrow \pi\pi$. Namely, while the factor $|P_{f \rightarrow a}(s)|^2$ has a sharp peak at the $K\bar{K}$ thresholds due to the $a_0(980)$ - $f_0(980)$ mixing, $|P_f(s)|^2$ has a relatively broad bump according to the decay width of $f_0(980)$. Therefore, the numerator of Eq. (26) becomes nearly comparable with the denominator momentarily at the $K\bar{K}$ thresholds, but when they are integrated the total amount of the numerator becomes only a few or less than a percent of that of the denominator in Eq. (25). In this study we compare theoretical values of the mixing intensity with the experimental one, which was obtained as the ratio of two branching fractions of J/ψ , $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0(980) \rightarrow \phi \pi\eta$ to $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi \pi\pi$, so we employ Eq. (25) to calculate the mixing intensity in the following.

TABLE I: Masses and coupling constants of the $a_0(980)$ and $f_0(980)$ resonances in the Flatte parametrization (9) determined from experimental data. Here we only show the central values except for the $K\bar{K}$ coupling constant. Coupling constants are given in the isospin basis.

$a_0(980)$			
Collaboration	M_a [MeV]	$\bar{g}_{aK\bar{K}}$ [GeV]	$\bar{g}_{a\pi\eta}$ [GeV]
CLEO [44]	998	3.97 ± 0.77	4.25
KLOE [45]	982.5	2.84 ± 0.41	2.46
CB [46]	987.4	2.94 ± 0.12	2.87
SND [47]	995	$5.93^{+10.54}_{-2.39}$	3.11
E852 [48]	1001	2.36 ± 0.13	2.47

$f_0(980)$			
Collaboration	M_f [MeV]	$\bar{g}_{fK\bar{K}}$ [GeV]	$\bar{g}_{f\pi\pi}$ [GeV]
CDF [49]	989.6	$4.02^{+1.01}_{-1.37}$	2.65
KLOE [50]	977.3	2.45 ± 0.17	1.21
Belle [51]	950	$4.07^{+0.76}_{-0.95}$	2.28
BES [52]	965	$5.80^{+0.22}_{-0.23}$	2.83
FOCUS [53]	957	$3.39^{+0.62}_{-0.76}$	2.15
SND [54]	969.8	$7.88^{+1.09}_{-0.86}$	3.19

D. Mixing intensity from experimental Flatte parameter sets

Since we have formulated the $a_0(980)$ - $f_0(980)$ mixing intensity in the previous section, we now would like to evaluate the $a_0(980)$ - $f_0(980)$ mixing intensity (25) with the Flatte parameters (9) determined from experimental data. Actually several collaborations reported the Flatte parameters for both the $a_0(980)$ and $f_0(980)$ resonances fitted to the experimental observations. In this study we employ parameters by CLEO [44], KLOE [45], CB [46], SND [47], and E852 [48] for $a_0(980)$, and by CDF [49], KLOE [50], Belle [51], BES [52], FOCUS [53], and SND [54] for $f_0(980)$. The parameter sets are listed in Table I. We note that the coupling constants in Table I are given in the isospin basis as in Eq. (10), and especially the $K\bar{K}$ coupling constants in the particle basis, \bar{g}_a and \bar{g}_f [see Eq. (4)], are evaluated with

$$\bar{g}_a = \frac{1}{\sqrt{2}}\bar{g}_{aK\bar{K}}, \quad \bar{g}_f = \frac{1}{\sqrt{2}}\bar{g}_{fK\bar{K}}, \quad (27)$$

where the factor $1/\sqrt{2}$ translates the coupling constants from the isospin basis ($\bar{g}_{aK\bar{K}}$ and $\bar{g}_{fK\bar{K}}$) into the particle basis. In this study we take into account the errors only for the $K\bar{K}$ coupling constants, which will strongly affect the $a_0(980)$ - $f_0(980)$ mixing intensity, while we take the central values for other parameters.

The numerical results of the $a_0(980)$ - $f_0(980)$ mixing intensity with all the combinations of the Flatte parameter sets are given in Table II. These values should be compared to the experimental value [28]:

$$\xi_{fa} = 0.60 \pm 0.20_{(\text{stat})} \pm 0.12_{(\text{sys})} \pm 0.26_{(\text{para})}\%, \quad (28)$$

$$\xi_{fa}|_{\text{upper limit}} = 1.1\% \quad (90\% \text{ C.L.}) \quad (29)$$

which was obtained as the ratio of two branching fractions of J/ψ , $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0(980) \rightarrow \phi\pi\eta$ to $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi\pi\pi$. It is remarkable that two thirds of the combinations of the Flatte parameter sets reproduce the experimental value with the errors (28) while only four combinations exceed the experimental upper limit of the mixing intensity (29). We also note that some of the parameter sets tend to lead to small or large mixing intensity. For instance, the $f_0(980)$ parameter set by FOCUS gives smaller mixing intensity, and the $a_0(980)$ parameter set by SND gives larger mixing intensity. Nevertheless, every parameter set can reproduce the experimental value of the mixing intensity with a suitable combination. In this sense we cannot rule out any parameter set in Table I with the experimental value of the $a_0(980)$ - $f_0(980)$ mixing intensity.

III. THE COMPOSITENESS

In this study we would like to give a way to extract more information on the structure of the $a_0(980)$ and $f_0(980)$ resonances. For this purpose, we introduce the compositeness, which corresponds to the amount of two-body states composing resonances as well as bound states. After a brief review of the so-called chiral unitary approach and compositeness in Sec. III A, we calculate the $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances in Sec. III B by using the Flatte parameter sets.

A. Chiral unitary approach and compositeness

In this subsection we briefly review the so-called chiral unitary approach, which provides scattering amplitudes of two pseudoscalar mesons [11–14] as well as a pseudoscalar meson and a baryon [55–60] from the coupled-channel unitarization of the interaction kernel taken from chiral Lagrangians. In this approach chiral interactions between two hadrons dynamically generate hadronic resonances in several channels from the meson-meson and meson-baryon degrees of freedom with successful reproductions of experimental observables. Then, in recent studies within the chiral unitary approach, structure of the dynamically generated states is intensively discussed in terms of compositeness [30–36], which corresponds to the amount of two-body states composing resonances as well as bound states. Here we also give the expression of the compositeness in the chiral unitary approach.

TABLE II: The $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} in percentage from the Flatte parameters in Table I with the errors for the $K\bar{K}$ coupling constants. The central value of the mixing intensity is shown in bold when it is consistent with the experimental value (28), in italic when out of the experimental errors, and with underline when above the upper limit (29).

$a_0(980)$	$f_0(980)$	CDF [49]	KLOE [50]	Belle [51]	BES [52]	FOCUS [53]	SND [54]
CLEO [44]		<i>0.21</i> $^{+0.30}_{-0.16}$	0.53 $^{+0.33}_{-0.23}$	0.26 $^{+0.30}_{-0.16}$	0.43 $^{+0.22}_{-0.17}$	<i>0.20</i> $^{+0.22}_{-0.13}$	0.73 $^{+0.72}_{-0.38}$
KLOE [45]		0.32 $^{+0.40}_{-0.23}$	0.81 $^{+0.41}_{-0.30}$	0.38 $^{+0.39}_{-0.23}$	0.65 $^{+0.26}_{-0.21}$	0.30 $^{+0.28}_{-0.18}$	<u>1.11</u> $^{+0.93}_{-0.51}$
CB [46]		0.26 $^{+0.24}_{-0.17}$	0.64 $^{+0.18}_{-0.15}$	0.31 $^{+0.22}_{-0.16}$	0.52 $^{+0.10}_{-0.09}$	<i>0.24</i> $^{+0.16}_{-0.12}$	0.89 $^{+0.50}_{-0.30}$
SND [47]		0.60 $^{+0.57}_{-0.49}$	<u>1.52</u> $^{+0.40}_{-0.91}$	0.70 $^{+0.47}_{-0.52}$	<u>1.22</u> $^{+0.20}_{-0.69}$	0.55 $^{+0.35}_{-0.40}$	<u>2.12</u> $^{+1.00}_{-1.38}$
E852 [48]		<i>0.19</i> $^{+0.19}_{-0.13}$	0.47 $^{+0.14}_{-0.12}$	<i>0.22</i> $^{+0.17}_{-0.12}$	0.39 $^{+0.08}_{-0.07}$	<i>0.18</i> $^{+0.12}_{-0.09}$	0.66 $^{+0.40}_{-0.23}$

In the chiral unitary approach, we solve the Bethe-Salpeter equation in an algebraic form so as to obtain a scattering amplitude of two pseudoscalar mesons

$$T_{ij}(s) = V_{ij}(s) + \sum_k V_{ik}(s)G_k(s)T_{kj}(s), \quad (30)$$

with channel indices i, j , and k , the Mandelstam variable s , the separable interaction kernel V to be fixed later, and the loop function G defined as

$$\begin{aligned} G_i(s) &\equiv i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_i^2 + i0} \frac{1}{(P-q)^2 - m_i'^2 + i0} \\ &= \int \frac{d^3q}{(2\pi)^3} \frac{\omega_i(\mathbf{q}) + \omega_i'(\mathbf{q})}{2\omega_i(\mathbf{q})\omega_i'(\mathbf{q})} \frac{1}{s - [\omega_i(\mathbf{q}) + \omega_i'(\mathbf{q})]^2 + i0}, \end{aligned} \quad (31)$$

where $P^\mu = (\sqrt{s}, \mathbf{0})$ and m_i and m_i' are masses of pseudoscalar mesons in channel i . In the second line we have performed the q^0 integral and $\omega_i(\mathbf{q}) \equiv \sqrt{m_i^2 + \mathbf{q}^2}$ and $\omega_i'(\mathbf{q}) \equiv \sqrt{m_i'^2 + \mathbf{q}^2}$ are the on-shell energies.

In this construction, a sufficiently strong attractive interaction with a coupling to an open channel can dynamically generate a resonance state, which appears as a pole of the scattering amplitude in the complex lower-half energy plane above the lowest threshold. The resonance pole is characterized by the pole position and the residue of the scattering amplitude as

$$T_{ij}(s) = \frac{g_i g_j}{s - s_{\text{pole}}} + T_{ij}^{\text{BG}}(s), \quad (32)$$

where g_i can be interpreted as the coupling constant of the resonance to the channel i , $\text{Re}\sqrt{s_{\text{pole}}} - 2\text{Im}\sqrt{s_{\text{pole}}}$ corresponds to the mass (width) of the resonance, and T_{ij}^{BG} is a background term which is regular at $s \rightarrow s_{\text{pole}}$. Then, in Refs. [30–36] the pole position and coupling constant are further translated into compositeness, which is defined as the two-body contribution to the normalization of the total wave function for the resonance. In our notations of the separable interaction and the loop function, the i -th channel two-body wave function for resonances generated with the Bethe-Salpeter equation (30)

is calculated as [36]

$$\tilde{\Psi}_i(\mathbf{q}) = \frac{g_i}{s_{\text{pole}} - [\omega_i(\mathbf{q}) + \omega_i'(\mathbf{q})]^2}, \quad (33)$$

and the compositeness is obtained as [30, 36]⁴

$$\begin{aligned} X_i &\equiv \int \frac{d^3q}{(2\pi)^3} \frac{\omega_i(\mathbf{q}) + \omega_i'(\mathbf{q})}{2\omega_i(\mathbf{q})\omega_i'(\mathbf{q})} [\tilde{\Psi}_i(\mathbf{q})]^2 \\ &= -g_i^2 \frac{dG_i}{ds}(s = s_{\text{pole}}), \end{aligned} \quad (34)$$

where the normalization factor $[\omega_i(\mathbf{q}) + \omega_i'(\mathbf{q})]/[2\omega_i(\mathbf{q})\omega_i'(\mathbf{q})]$ guarantees the Lorentz invariance of the integral and in the last line the integral is transformed into the derivative of the loop function G_i (31). We note that the compositeness is not an observable and hence is a model dependent quantity. We also note that the derivative of the loop function does not diverge for meson-meson states in contrast to the loop function itself, but one has to treat consistently the loop function and its derivative, i.e., one has to use the same regularization for both the loop function and its derivative. On the other hand, in order to measure the fraction of the bare state contribution rather than the two-body state involved, we introduce the elementariness Z , which corresponds to the field renormalization constant intensively discussed in the 1960s [61–64]. The elementariness measures all contributions which cannot be responsible for the hadronic two-body component involved. For instance, compact $q\bar{q}$ and $qq\bar{q}\bar{q}$ states contribute to the elementariness. The expression of the elementariness in our notations is obtained in Ref. [36]

⁴ For the correct normalization of the resonance wave function we do not calculate the absolute value squared but the complex number squared of $\tilde{\Psi}(\mathbf{q})$. Moreover, the bra and ket vectors of the resonance state should be $\langle\Psi^*|$ and $|\Psi\rangle$, respectively, so as to obtain the correct normalization, $\langle\Psi^*|\Psi\rangle = 1$ (see Refs. [34, 36] for details).

as

$$Z = - \sum_{i,j} g_j g_i \left[G_i \frac{dV_{ij}}{ds} G_j \right]_{s=s_{\text{pole}}}. \quad (35)$$

We note that in general both the compositeness X_i and elementariness Z take complex values for a resonance state and hence one cannot interpret the compositeness (elementariness) as the probability to observe a two-body (bare-state) component inside the resonance. However, a striking property of the compositeness and elementariness is that sum of them coincides with the normalization of the total wave function for the resonance $|\Psi\rangle$ and is exactly unity [36]:

$$\begin{aligned} \langle \Psi^* | \Psi \rangle &= \sum_i X_i + Z \\ &= - \sum_{i,j} g_j g_i \left[\delta_{ij} \frac{dG_i}{ds} + G_i \frac{dV_{ij}}{ds} G_j \right]_{s=s_{\text{pole}}} = 1, \end{aligned} \quad (36)$$

where the condition of the correct normalization as unity is guaranteed by a generalized Ward identity proved in Ref. [65]. Based on this normalization, we propose to interpret the compositeness X_i and elementariness Z for a certain class of resonances on the basis of the similarity to the bound state case. Namely, if the compositeness X_1 approaches unity with a small imaginary part while X_i ($i \neq 1$) and Z negligibly contribute to the normalization (36), the system can be interpreted to be dominated by the two-body component in channel 1, since the resonance wave function is considered to be similar to that of the bound state dominated by the channel 1. In this sense, $|X_1| \sim 1$ is a necessary condition for the molecular picture in channel 1. In another case, if $|X_i|$ is much smaller than unity, the system contains negligible i -channel two-body component.

In order to examine the chiral unitary approach and compositeness, let us consider scatterings of s -wave two pseudoscalar mesons which couples to the $a_0(980)$ and $f_0(980)$ resonances. We here assume the isospin symmetry and introduce five channels labeled by the indices $i = 1, \dots, 5$ in the order K^+K^- , $K^0\bar{K}^0$, $\pi^+\pi^-$, $\pi^0\pi^0$, and $\pi^0\eta$. The interaction kernel $V_{ij} = V_{ji}$ is taken from the leading-order chiral Lagrangian as

$$\begin{aligned} V_{11} &= 2V_{12} = 2V_{13} = 2\sqrt{2}V_{14} \\ &= V_{22} = 2V_{23} = 2\sqrt{2}V_{24} = V_{33} = -\frac{s}{2f^2}, \\ V_{15} &= -V_{25} = -\frac{3s - 4m_K^2}{4\sqrt{3}f^2}, \\ V_{34} &= -\frac{s - m_\pi^2}{\sqrt{2}f^2}, \\ V_{35} &= V_{45} = 0, \\ V_{44} &= \frac{3}{2}V_{55} = -\frac{m_\pi^2}{2f^2}, \end{aligned} \quad (37)$$

TABLE III: Properties of the $a_0(980)$ and $f_0(980)$ resonances in the chiral unitary approach. For later convenience we show the values in the particle basis.

	$a_0(980)$	$f_0(980)$
$\sqrt{s_{\text{pole}}}$	$977 - 56i$ MeV	$983 - 21i$ MeV
$g_{K^+K^-}$	$3.31 + 0.28i$ GeV	$2.97 + 0.89i$ GeV
$g_{K^0\bar{K}^0}$	$-3.31 - 0.28i$ GeV	$2.97 + 0.89i$ GeV
$g_{\pi^+\pi^-}$	—	$-0.29 + 1.28i$ GeV
$g_{\pi^0\pi^0}$	—	$-0.20 + 0.90i$ GeV
$g_{\pi^0\eta}$	$2.99 - 0.85i$ GeV	—
$X_{K^+K^-}$	$0.17 - 0.15i$	$0.35 - 0.05i$
$X_{K^0\bar{K}^0}$	$0.17 - 0.15i$	$0.35 - 0.05i$
$X_{\pi^+\pi^-}$	—	$0.01 + 0.01i$
$X_{\pi^0\pi^0}$	—	$0.01 + 0.00i$
$X_{\pi^0\eta}$	$-0.07 + 0.12i$	—
Z	$0.73 + 0.18i$	$0.28 + 0.10i$

with the pion decay constant f . We note that we have multiplied the interaction kernel (37) in the case of $\pi^0\pi^0$ states by $1/\sqrt{2}$ compared to the expression given in, *e.g.*, Ref. [24], thus a naïve unitarization in Eq. (30) with the interaction kernel (37) can give a correct normalization for intermediate states of identical particles. On the other hand, for the loop function we employ a three-dimensional cut-off as

$$\begin{aligned} G_i(s; q_{\text{max}}) &= \int \frac{d^3q}{(2\pi)^3} \frac{\omega_i(\mathbf{q}) + \omega'_i(\mathbf{q})}{2\omega_i(\mathbf{q})\omega'_i(\mathbf{q})} \frac{\theta(q_{\text{max}} - |\mathbf{q}|)}{s - [\omega_i(\mathbf{q}) + \omega'_i(\mathbf{q})]^2 + i0}, \end{aligned} \quad (38)$$

where $\theta(x)$ is the Heaviside step function.

Now we solve the Bethe-Salpeter equation (30) with the isospin symmetric masses and parameters $q_{\text{max}} = 1.075$ GeV and $f = 93.0$ MeV, which are chosen so as to generate two poles which correspond to the $a_0(980)$ and $f_0(980)$ resonances, respectively, in the complex s plane of the scattering amplitude. The pole positions, coupling constants, compositeness, and elementariness of the two resonances are listed in Table III. We note that for the evaluations of the compositeness and elementariness we use the same sharp cut-off q_{max} for the loop function and its derivative. As a result, the $K\bar{K}$ compositeness for the resonance, $X_{K\bar{K}}$, can be obtained by summing up the $i = 1$ (K^+K^-) and 2 ($K^0\bar{K}^0$) contributions as

$$X_{K\bar{K}} \equiv - \sum_{i=1}^2 g_i^2 \frac{dG_i}{ds}(s = s_{\text{pole}}; q_{\text{max}}), \quad (39)$$

which results in $0.34 - 0.30i$ for $a_0(980)$ and $0.70 - 0.11i$ for $f_0(980)$. Since the real part dominates the sum rule (36) while the imaginary part is negligible, the $K\bar{K}$ compositeness for $f_0(980)$ indicates a large $K\bar{K}$ component inside it, on the basis of the similarity to the

bound state case. This finding is consistent with a model-independent analysis in Ref. [16]. The $K\bar{K}$ compositeness for $a_0(980)$ implies a nonnegligible $K\bar{K}$ component inside it, but we cannot clearly conclude the structure due to its large imaginary part. The difference of the structure of $a_0(980)$ and $f_0(980)$ may originate from the fact that the strength of the leading-order $K\bar{K}(I=0)$ interaction from chiral perturbation theory [see Eq. (37)] is three times larger than that with $I=1$:

$$V_{K\bar{K}(I=0)} = -\frac{3s}{4f^2}, \quad V_{K\bar{K}(I=1)} = -\frac{s}{4f^2}. \quad (40)$$

In addition, the absolute values of the $\pi\eta$ and $\pi\pi$ compositeness are much smaller than unity, so both the $\pi\eta$ component inside $a_0(980)$ and the $\pi\pi$ component inside $f_0(980)$ are negligible.

At the end of this subsection we emphasize that, although the compositeness is not an observable, we can evaluate it from experimental observables via appropriate models. In the following we employ the expression in Eq. (39) for the $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances. We will take the cut-off $q_{\max} \rightarrow \infty$ for the derivative of the loop function G_i ; the use of finite cut-off $q_{\max} \sim 1$ GeV will give only several percent change of the value of the compositeness.

B. $K\bar{K}$ compositeness from experimental Flatte parameter sets

As we have seen in the previous subsection, the $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ mesons (39) can be determined with their pole positions and residues of the scattering amplitudes. Here we adopt the Flatte parametrization without the mixing in Eq. (9), and we calculate the pole positions, residues, and compositeness for the scalar mesons from experimentally fitted parameters in Table I. Namely, the propagator of the scalar meson A ($A = a, f$), $1/D_A(s)$, brings a pole in the scattering amplitude of T_{ij}^A :

$$T_{ij}^A = \frac{\bar{g}_{Ai}\bar{g}_{Aj}}{D_A(s)} + (\text{regular at } s = s_A), \quad (41)$$

where s_A is the pole position of $1/D_A(s)$ and $\bar{g}_{Ai}\bar{g}_{Aj}$ is multiplied so as to describe the scattering i to j . Therefore, compared to Eq. (32) the residue at the resonance pole of the scattering amplitude T^A can be evaluated as

$$g_A^2 = \bar{g}_A^2 R_A, \quad (42)$$

$$R_A \equiv \text{Res}[1/D_A(s); s_A] = \lim_{s \rightarrow s_A} \frac{s - s_A}{D_A(s)}. \quad (43)$$

As a consequence, the $K\bar{K}$ compositeness for the resonance A , X_A , can be obtained by summing up the $i = 1$

TABLE IV: Pole positions and compositeness from the Flatte parameters given in Table I. Here we only show the central values. Compositeness is given in the isospin basis.

$a_0(980)$			
Collaboration	$\sqrt{s_a}$ [MeV]	X_a	$X_{\pi\eta}$
CLEO [44]	$1022 - 70i$	$0.09 - 0.22i$	$-0.16 + 0.05i$
KLOE [45]	$994 - 26i$	$0.15 - 0.17i$	$-0.05 + 0.03i$
CB [46]	$1000 - 36i$	$0.12 - 0.17i$	$-0.07 + 0.03i$
SND [47]	$1005 - 5i$	$0.68 - 0.51i$	$-0.05 - 0.01i$
E852 [48]	$1007 - 28i$	$0.07 - 0.15i$	$-0.06 + 0.03i$

$f_0(980)$			
Collaboration	$\sqrt{s_f}$ [MeV]	X_f	$X_{\pi\pi}$
CDF [49]	$1010 - 30i$	$0.21 - 0.30i$	$-0.03 - 0.01i$
KLOE [50]	$985 - 10i$	$0.21 - 0.12i$	$-0.01 - 0.00i$
Belle [51]	$983 - 27i$	$0.29 - 0.17i$	$-0.02 - 0.00i$
BES [52]	$1000 - 18i$	$0.50 - 0.34i$	$-0.02 - 0.01i$
FOCUS [53]	$981 - 28i$	$0.21 - 0.14i$	$-0.02 - 0.00i$
SND [54]	$1001 - 7i$	$0.80 - 0.31i$	$-0.01 - 0.01i$

(K^+K^-) and 2 ($K^0\bar{K}^0$) contributions as

$$\begin{aligned} X_A &= -g_A^2 \sum_{i=1}^2 \frac{dG_i}{ds}(s = s_A; \infty) \\ &= -\bar{g}_A^2 R_A \sum_{i=1}^2 \frac{dG_i}{ds}(s = s_A; \infty). \end{aligned} \quad (44)$$

This is the formula to calculate the $K\bar{K}$ compositeness of the scalar mesons from the Flatte parametrization without the mixing. In a similar manner we can calculate the $\pi\pi$ and $\pi\eta$ compositeness for the scalar mesons.

Now we calculate the pole positions and compositeness of the $a_0(980)$ and $f_0(980)$ scalar mesons from the parameter sets in Table I. The numerical results are listed in Table IV and plotted in Fig. 3. In the table we show only the central values, while we take into account the errors for the $K\bar{K}$ coupling constants in the figure. All of the pole positions in Table IV exist in the physical Riemann sheet of the $K\bar{K}$ channel.

As one can see from Table IV and Fig. 3, we obtain the complex compositeness in every parameter sets since $a_0(980)$ and $f_0(980)$ are resonance states. For the $K\bar{K}$ compositeness of $a_0(980)$, the parameters do not give large absolute value of the $K\bar{K}$ compositeness comparable to unity except for the SND parameter, which however has a large error bar as seen in Fig. 3. The absolute value $|X_a| \sim 0.2$ in other parameter sets could imply a small but nonnegligible $K\bar{K}$ component inside $a_0(980)$, but at present we do not clearly interpret the $K\bar{K}$ compositeness for $a_0(980)$. On the other hand, for $f_0(980)$ two of the parameter sets (BES and SND) imply large absolute value of the $K\bar{K}$ component with, say $|X_f| > 0.6$. Especially it is worth noting that in the BES analysis

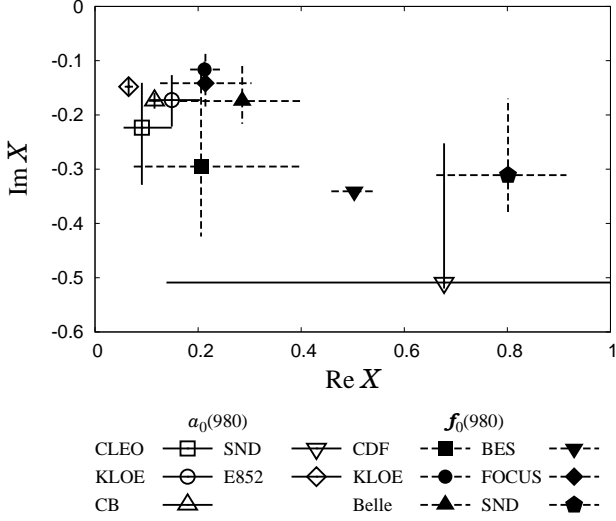


FIG. 3: The $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances from the the Flatte parameters in Table I with the errors for the $K\bar{K}$ coupling constants. Open (filled) symbols with solid (dashed) lines represent the $K\bar{K}$ compositeness for $a_0(980)$ [$f_0(980)$].

the authors fitted the $K\bar{K}$ spectrum as well as the $\pi\pi$ spectrum, which leads to a small error coming from the $K\bar{K}$ coupling constant. We note that the tendency for $f_0(980)$ supports the result in the chiral unitary approach (see Table III and also Ref. [36, 66]).

The tendency for $f_0(980)$ is similar to the findings in Ref. [16], which suggested that $f_0(980)$ should be a $K\bar{K}$ molecular state to a large degree. However, although both results in the present study and in Ref. [16] are obtained with the experimental Flatte parameters and similar formulations, a big difference is that in Ref. [16] they defined “compositeness” in terms of the spectral density as a real value even for the $a_0(980)$ and $f_0(980)$ resonances. This treatment should be valid effectively only when the resonance has a narrow width and the imaginary part of the compositeness is enough small (see also Ref. [34]). Otherwise, the correct normalization (36) will be lost. However, our result gives a nonnegligible imaginary part of the compositeness from the Flatte parameters, which means that the treatment in Ref. [16] should be reexamined. In other words, the value calculated in Ref. [16] should not be compared with unity, since the correct normalization of the resonance wave function should be lost.

We also note that all of the absolute values of the $\pi\eta$ and $\pi\pi$ compositeness for the $a_0(980)$ and $f_0(980)$ resonances, respectively, are small compared to unity. This strongly indicates that the $a_0(980)$ and $f_0(980)$ resonances are not the $\pi\eta$ and $\pi\pi$ molecular states, respectively.

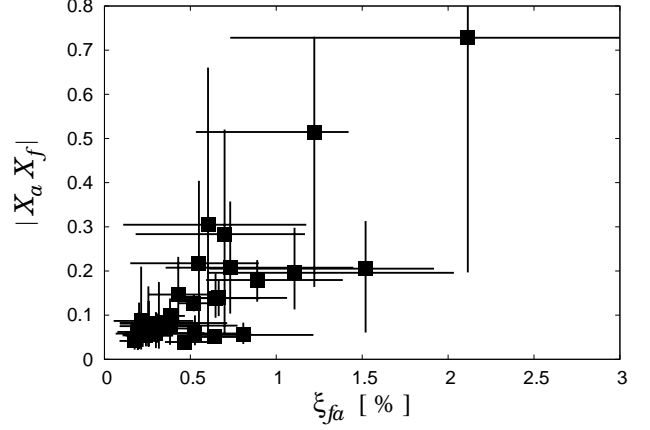


FIG. 4: Scatter plot of the absolute value of the product of the $K\bar{K}$ compositeness for the $a_0(980)$ and $f_0(980)$ resonances, $|X_a X_f|$, with respect to the $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} . Data points are obtained from the Flatte parameters given in Table I with the errors for the $K\bar{K}$ coupling constants.

IV. CONSTRAINT ON THE $K\bar{K}$ COMPOSITENESS FROM THE $a_0(980)$ - $f_0(980)$ MIXING INTENSITY

A. Relation between the $a_0(980)$ - $f_0(980)$ mixing intensity and the $K\bar{K}$ compositeness from experimental Flatte parameter sets

Now that we have formulated both the $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} and the compositeness X , we would like to investigate a relation between them for the $a_0(980)$ and $f_0(980)$ resonances. To this end we first mention that the mixing amplitude $\Lambda(s)$ (3) is proportional to the product of the coupling constants, $\bar{g}_a \bar{g}_f$. This indicates that, when the product of the coupling constants $\bar{g}_a \bar{g}_f$ and hence the mixing amplitude $\Lambda(s)$ are sufficiently small, the mixing intensity behaves in power of

$$\xi_{fa} \sim |\Lambda|^2 \sim |\bar{g}_a \bar{g}_f|^2 \propto |X_a X_f|. \quad (45)$$

Therefore, we expect that the mixing intensity ξ_{fa} is proportional to the absolute value of the product of the $K\bar{K}$ compositeness of $a_0(980)$ and $f_0(980)$, $|X_a X_f|$, for small $K\bar{K}$ coupling constants.

In order to examine the behavior in Eq. (45), we plot in Fig. 4 the absolute value of the product of the $K\bar{K}$ compositeness for the $a_0(980)$ and $f_0(980)$ resonances, $|X_a X_f|$, with respect to the $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} by using the Flatte parameters in Table I. As one can see from Fig. 4, although a clear proportional connection $\xi_{fa} \propto |X_a X_f|$ is not observed, there is a tendency that the product $|X_a X_f|$ increases as the mixing intensity ξ_{fa} increases. From the experimental upper limit of the mixing intensity (29), we expect that the product of the compositeness has a upper bound as $|X_a X_f| \lesssim 0.4$. This upper bound implies

that the $a_0(980)$ and $f_0(980)$ resonances cannot be simultaneously $K\bar{K}$ molecular states, since the condition $|X_a| \sim |X_f| \sim 1$ cannot satisfy $|X_a X_f| \lesssim 0.4$.

We note that the above discussion is based on the Flatte parameters in Table I. Therefore, in order to confirm this relation between the mixing intensity and the absolute value of the $K\bar{K}$ compositeness for $a_0(980)$ and $f_0(980)$, we have to calculate them with more general parameter sets, especially for the $a_0(980)$ - and $f_0(980)$ - $K\bar{K}$ coupling constants, which are responsible for both the mixing intensity and their $K\bar{K}$ compositeness. This is the task in the next subsection.

B. Confirmation of the relation

1. Strategy

Now we construct a relation between the $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} and the compositeness X for the $a_0(980)$ and $f_0(980)$ resonances. Our strategy is summarized as follows. First, we fix four parameters M_a , M_f , $\bar{g}_{a\pi\eta}$, and $\bar{g}_{f\pi\pi}$ in some appropriate approaches. Then we generate the $a_0(980)$ - and $f_0(980)$ - $K\bar{K}$ coupling constants, \bar{g}_a and \bar{g}_f , respectively, which are responsible for both the mixing intensity ξ_{fa} (25) and their $K\bar{K}$ compositeness, to evaluate simultaneously the mixing intensity and their $K\bar{K}$ compositeness. With this approach we can give a more general constraint on the $K\bar{K}$ structure of $a_0(980)$ and $f_0(980)$ regardless of the details of the $a_0(980)$ - and $f_0(980)$ - $K\bar{K}$ coupling constants.

For the $K\bar{K}$ compositeness we employ the model in Sec. III, and evaluate it with the following expression:

$$X_A = -g_A^2 \sum_{i=1}^2 \frac{dG_i}{ds}(s = s_A; \infty), \quad A = a, f. \quad (46)$$

In order that we can calculate the $K\bar{K}$ compositeness with not only small but also large mixing amplitude, the pole position s_A for the resonance A is extracted as that of the propagator $P_A(s)$ rather than the propagator without mixing, $1/D_A(s)$. We note that the coupling constant g_A in the expression of the compositeness should be evaluated as a residue of the resonance pole position [see Eq. (32)], and hence it differs from \bar{g}_A by the residue of the propagator $P_A(s)$. Namely, in a similar manner to the discussion in Sec III B, taking into account the residue of the propagator $P_A(s)$ as

$$g_A^2 = \bar{g}_A^2 R'_A, \quad R'_A \equiv \lim_{s \rightarrow s_A} (s - s_A) P_A(s), \quad (47)$$

we can calculate the compositeness as

$$X_A = -\bar{g}_A^2 R'_A \sum_{i=1}^2 \frac{dG_i}{ds}(s = s_A; \infty). \quad (48)$$

Although the compositeness X_A is in general complex for resonance states, in this study we use Eq. (48) so as

to evaluate the absolute value of the compositeness $|X_A|$ from the coupling constant \bar{g}_A . Therefore, in our strategy we will obtain a relation between the mixing intensity and the absolute value of the $K\bar{K}$ compositeness for $a_0(980)$ and $f_0(980)$. We emphasize that the absolute value of the compositeness cannot be interpreted as a probability to find a two-body molecular state but it will be an important piece of information on the structure of the scalar mesons when compared with unity. For instance, $|X_A| \sim 1$ is a necessary condition for the $K\bar{K}$ molecular picture of the meson A , while $|X_A| \ll 1$ indicates that the meson A has a negligible $K\bar{K}$ molecular component.

2. Relation between the $a_0(980)$ - $f_0(980)$ mixing intensity and the $K\bar{K}$ compositeness

We now fix the four parameters as rough averages of the Flatte parameters listed in Table I:

$$\begin{aligned} M_a &= 990 \text{ MeV}, & \bar{g}_{a\pi\eta} &= 3.0 \text{ GeV}, \\ M_f &= 970 \text{ MeV}, & \bar{g}_{f\pi\pi} &= 2.4 \text{ GeV}, \end{aligned} \quad (49)$$

to construct a relation between the $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances and their mixing intensity ξ_{fa} . Since we expect that the mixing intensity behaves as in Eq. (45) for small $K\bar{K}$ coupling constants, we investigate a relation between the mixing intensity ξ_{fa} and the absolute value of the product of the $K\bar{K}$ compositeness of $a_0(980)$ and $f_0(980)$, $|X_a X_f|$. Here we employ the Monte-Carlo method and generate values of the coupling constants \bar{g}_a and \bar{g}_f independently from random numbers. In the Monte-Carlo method we take the range of the $K\bar{K}$ coupling constants as $[0 \text{ GeV}, 6 \text{ GeV}]$ both for \bar{g}_a and \bar{g}_f , which covers the values of the coupling constants $\bar{g}_{aK\bar{K}} (= \sqrt{2}\bar{g}_a)$ and $g_{fK\bar{K}} (= \sqrt{2}\bar{g}_f)$ listed in Table I.

In Fig. 5 we show a scatter plot for the absolute value of the product of the $K\bar{K}$ compositeness of $a_0(980)$ and $f_0(980)$, $|X_a X_f|$, with respect to the mixing intensity ξ_{fa} , with various values of the coupling constants \bar{g}_a and \bar{g}_f from random numbers. As one can see from Fig. 5, although a proportional connection $\xi_{fa} \propto |X_a X_f|$ is not observed, no Monte-Carlo data point exists in the upper-left region of the plot, which implies that there is an upper limit of allowed $|X_a X_f|$ for each value of ξ_{fa} . In fact we can check the existence of this upper limit by sweeping the values of \bar{g}_a and \bar{g}_f independently in appropriate ranges, say $[0 \text{ GeV}, 6 \text{ GeV}]$, and the result of the upper limit of $|X_a X_f|$ for each ξ_{fa} is plotted as a solid line in Fig. 5. We note that the upper limit of the allowed $|X_a X_f|$ behaves like $|X_a X_f|_{\text{upper limit}} \sim \xi_{fa}$. This result means that, if we observe smaller value of the mixing intensity ξ_{fa} , the product of the compositeness $|X_a X_f|$ also becomes smaller.

We then discuss the compositeness of the scalar mesons with the experimental value of the mixing intensity (28) and (29), which was obtained as the ratio of two branching fractions of J/ψ , $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0(980) \rightarrow$

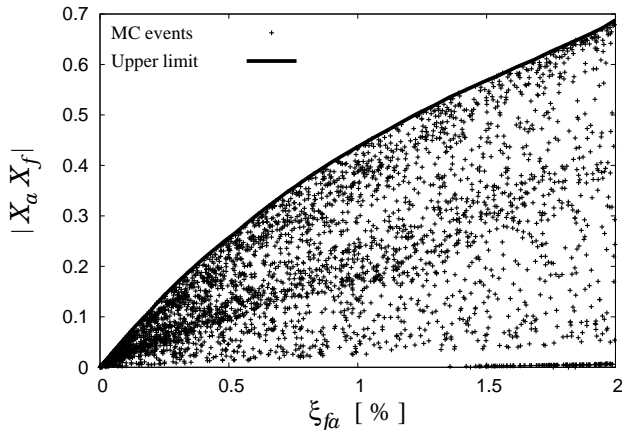


FIG. 5: Scatter plot of the absolute value of the product of the $K\bar{K}$ compositeness for the $a_0(980)$ and $f_0(980)$ resonances, $|X_a X_f|$, with respect to the $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} . In the plot, number of data in the Monte-Carlo method (MC events) amounts to $\sim 4 \times 10^3$. We also show an upper limit of $|X_a X_f|$ for each value of ξ_{fa} by the solid line.

$\phi\pi\eta$ to $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi\pi\pi$. This experimental value, together with the upper limit of $|X_a X_f|$ for each value of the mixing intensity ξ_{fa} shown in Fig. 5, can constrain the structure of $a_0(980)$ and $f_0(980)$ through their compositeness. For instance, with Fig. 5, $\xi_{fa}|_{\text{upper limit}} = 1.1\%$ gives a constraint $|X_a X_f| < 0.47$. With this constraint, we confirm that both the $a_0(980)$ and $f_0(980)$ resonances are simultaneously $K\bar{K}$ molecular states is questionable, since the condition $|X_a| \sim |X_f| \sim 1$ is out of the allowed region $|X_a X_f| < 0.47$.

The experimental value (28) and (29) constrains not only the value of the product $|X_a X_f|$ but also the allowed region for $|X_a|$ and $|X_f|$ in the $|X_a|$ - $|X_f|$ plane. Here we show in Fig. 6 the allowed region for $|X_a|$ and $|X_f|$ in the $|X_a|$ - $|X_f|$ plane calculated by sweeping the values of \bar{g}_a and \bar{g}_f independently. In Fig. 6 the shaded area corresponds to the allowed region within the experimental value (28) including errors, and the solid line to the upper limit from the experimental value (29), $\xi_{fa} = 1.1\%$. The region above the solid line in Fig. 6 inevitably leads to the mixing intensity $\xi_{fa} > 1.1\%$ and hence excluded. As one can see, the experimental value of the mixing intensity ξ_{fa} does not allow the region of $|X_a| \sim |X_f| \sim 1$, thus the statement that both the $a_0(980)$ and $f_0(980)$ resonances are simultaneously $K\bar{K}$ molecular states is questionable. In fact, this consequence was already implied in Ref. [28], in which the authors showed that the experimental mixing intensity disfavored the predicted value for $a_0(980)$ and $f_0(980)$ as $K\bar{K}$ molecules. We here note that conditions that one of the scalar mesons has large degree of the $K\bar{K}$ molecule, such as $|X_a| = 0.1$ and $|X_f| = 0.7$ or $|X_a| = 0.8$ and $|X_f| = 0.3$, are not forbidden by the experimental value of the mixing intensity. Especially, the band in Fig. 6 show $|X_f| \gtrsim 0.3$ regardless of the value of $|X_a|$, which might indicate a nonnegligible degree of the

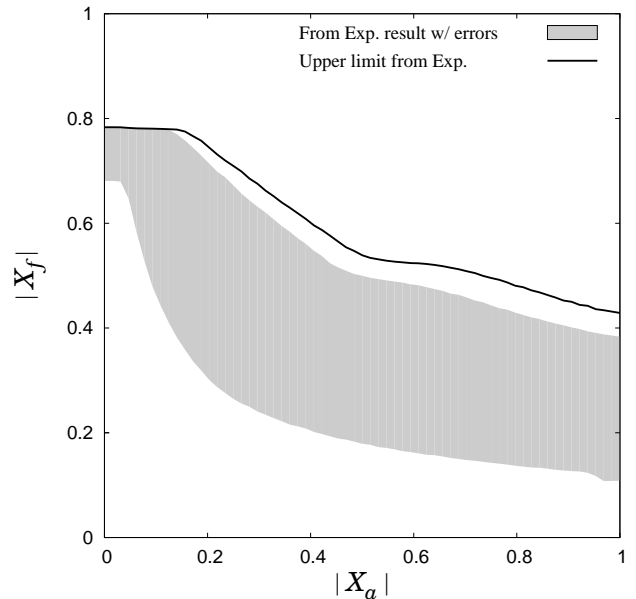


FIG. 6: Allowed region for the absolute values of compositeness $|X_a|$ and $|X_f|$ constrained by the experimental value of the $a_0(980)$ - $f_0(980)$ mixing intensity (28) and (29). The shaded area corresponds to the allowed region within the experimental value (28) including errors, and the solid line to the upper limit of the experimental value $\xi_{fa} = 1.1\%$ (29).

$K\bar{K}$ molecule for the $f_0(980)$ resonance. Moreover, the experimental mixing intensity does not disfavor the condition that both $a_0(980)$ and $f_0(980)$ have nonnegligible $K\bar{K}$ components with, for instance, $|X_a| = |X_f| = 0.4$.

V. DISCUSSION

So far we have considered the $a_0(980)$ - $f_0(980)$ mixing and have given a constraint on the $K\bar{K}$ compositeness for the $a_0(980)$ and $f_0(980)$ resonances from an established relation between the $K\bar{K}$ compositeness and the $a_0(980)$ - $f_0(980)$ mixing intensity. As a result, the experimental value of the mixing intensity implies that the $a_0(980)$ and $f_0(980)$ resonances cannot be simultaneously $K\bar{K}$ molecular states. The $a_0(980)$ - $f_0(980)$ mixing, however, cannot answer the question that whether one of $a_0(980)$ and $f_0(980)$ is a $K\bar{K}$ molecular state or not. In order to solve this problem experimentally, we have to call for other experimental data on these resonances.

One possible way to determine the structure of each scalar meson is to evaluate the compositeness. Actually Eq. (34) indicates that compositeness for each resonance can be evaluated from the pole position and the coupling constant as the residue at the pole position. From this point of view, in Sec. IIIB we have calculated the $K\bar{K}$ compositeness of the scalar mesons $a_0(980)$ and $f_0(980)$ described with the Flatté parameters in Table I. From the results shown in Table IV and in Fig. 3, we have found

that two of the parameter sets for $f_0(980)$ imply large absolute value of the $K\bar{K}$ component with, say $|X_f| > 0.6$, and the parameters for $a_0(980)$ lead to small but non-negligible $K\bar{K}$ compositeness $|X_a| \sim 0.2$. Nevertheless, in order to conclude the structure of the scalar mesons more strictly, we have to determine the pole positions and coupling constants more precisely in experiments. Especially it will be important to fit the $K\bar{K}$ spectrum as well as the $\pi\eta/\pi\pi$ spectrum, which will lead to a small error of the $K\bar{K}$ compositeness coming from the $K\bar{K}$ coupling constant, as done in the BES analysis in Ref. [52].

Another approach to determine the structure of the scalar mesons is to measure their spatial size, since a hadronic molecule can be a spatially extended object due to the absence of strong quark confining force. In fact, the spatial size of exotic hadron candidates was theoretically measured in a meson-meson and meson-baryon scatterings in, *e.g.*, Refs. [65–68], and it was found that $f_0(980)$ and $\Lambda(1405)$, whose $K\bar{K}$ and $\bar{K}N$ compositeness are close to unity, respectively, has spatial size exceeding largely the typical hadronic scale $\lesssim 0.8$ fm.

The internal structure of the scalar mesons could be also investigated by the ϕ radiative decays into $f_0(980)$ and $a_0(980)$, since $\phi \rightarrow f_0(980)\gamma$ and $a_0(980)\gamma$ are electric dipole decays and hence the widths should reflect spatial sizes of the scalar mesons [69–76]. The couplings of the scalar mesons to two photons are also sensitive to the structure of the scalar mesons and have been investigated in, *e.g.*, Refs. [77–81]. The measurements of the ϕ radiative decays and the two-photon couplings support the multiquark picture for the scalar mesons $f_0(980)$ and $a_0(980)$ [81].

In addition, high-energy reactions will be useful for determining internal structure of exotic hadron candidates since quarks and gluons are appropriate degrees of freedom at high energies. In this context, the generalized parton distributions and the generalized distribution amplitudes (GDAs) can be used to clarify internal configurations of exotic hadrons, especially $f_0(980)$ and $a_0(980)$ [82] by the GDAs in two-photon reactions [$\gamma\gamma^* \rightarrow AA$, $A = f_0(980), a_0(980)$]. Next, asymptotic scaling behavior of the production cross sections can be a guide to determine internal quark configurations of the exotic hadrons, such as $\Lambda(1405)$, due to the constituent-counting rule in perturbative QCD [83]. Fragmentation functions of exotic hadrons could also provide a clue for finding their internal configurations by using characteristic differences between favored and disfavored fragmentations [84], which could be measured at KEKB. Finally, the possibility to extract the hadron structure from the production yield in heavy ion collisions [85, 86] is also interesting, since one can distinguish hadronic molecules, compact exotic states, and ordinary quark configurations from the production yield.

VI. CONCLUSION

In this study we investigated the structure of the $a_0(980)$ and $f_0(980)$ resonances with the $a_0(980)$ - $f_0(980)$ mixing. Since the $a_0(980)$ - $f_0(980)$ mixing takes place through the difference of the thresholds of the charged and neutral $K\bar{K}$ pairs, the mixing should be sensitive to the $K\bar{K}$ components inside the scalar mesons. Actually the $a_0(980)$ - $K\bar{K}$ and $f_0(980)$ - $K\bar{K}$ coupling constants reflect the $K\bar{K}$ structure of the $a_0(980)$ and $f_0(980)$ resonances, respectively, and the mixing amplitude is proportional to the two coupling constants. The key quantity to connect the $a_0(980)$ - and $f_0(980)$ - $K\bar{K}$ coupling constants to their structure is compositeness, which is defined as the two-body composite part of the normalization of the total wave function and corresponds to the amount of two-body states composing resonances as well as bound states.

The $a_0(980)$ - $f_0(980)$ mixing intensity was defined in the same manner as the analysis by the BES experiment in Ref. [28], where the ratio of two branching fractions of J/ψ , $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0(980) \rightarrow \phi\pi\eta$ to $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi\pi\pi$, was evaluated. For the $a_0(980) \leftrightarrow f_0(980)$ mixing amplitude, we employed three Feynman diagrams; the leading-order contribution from the K^+K^- and $K^0\bar{K}^0$ loops, sum of which converges and becomes model independent except for the coupling constants, and a subleading contribution of a soft photon exchange in the K^+K^- loop. We took the Flatte parametrization for the $a_0(980)$ and $f_0(980)$ propagators. In this construction, when we appropriately fix the parameters of $a_0(980)$ and $f_0(980)$ including the $a_0(980)$ - and $f_0(980)$ - $K\bar{K}$ coupling constants, we can calculate the $a_0(980)$ - $f_0(980)$ mixing intensity ξ_{fa} . By using the existing Flatte parameter sets from experimental analyses, we found that two thirds of the combinations of the Flatte parameter sets reproduce the experimental value with the errors in Ref. [28] while only four combinations exceed the experimental upper limit of the mixing intensity.

From the same Flatte parameters we also calculated the $K\bar{K}$ compositeness for $a_0(980)$ and $f_0(980)$, X_a and X_f , respectively. Although the compositeness with the correct normalization becomes complex in general for resonance states, we found that two of the Flatte parameter sets for $f_0(980)$ give large absolute value of the $K\bar{K}$ component with, say $|X_f| > 0.6$, and the parameters for $a_0(980)$ lead to small but nonnegligible $K\bar{K}$ compositeness $|X_a| \sim 0.2$.

Next, combining the two results on the $a_0(980)$ - $f_0(980)$ mixing intensity and on their $K\bar{K}$ compositeness from the existing Flatte parameters, we found a relation between the mixing intensity and the absolute value of the product of the compositeness, $|X_a X_f|$, from experiments. This relation was confirmed by generating the values of the $a_0(980)$ - and $f_0(980)$ - $K\bar{K}$ coupling constants randomly, which are responsible for both the mixing intensity and the $K\bar{K}$ compositeness. As a result, we found

an upper limit of allowed $|X_a X_f|$ for each value of ξ_{fa} , which behaves like $|X_a X_f|_{\text{upper limit}} \sim \xi_{fa}$. Especially the result suggests that a small mixing intensity ξ_{fa} directly indicates a small value of $|X_a X_f|$. Then, by using the $a_0(980)$ - $f_0(980)$ mixing intensity recently observed in the BES experiment, we constrained the allowed region of the $K\bar{K}$ compositeness $|X_a|$ and $|X_f|$ in the $|X_a|$ - $|X_f|$ plane. We found that the region $|X_a| \sim |X_f| \sim 1$ is not preferred, which implies that the $a_0(980)$ and $f_0(980)$ resonances cannot be simultaneously $K\bar{K}$ molecular states. However, the analysis does not rule out possibilities that one of the scalar mesons has large degree of the $K\bar{K}$ molecule. Especially, we obtained $|X_f| \gtrsim 0.3$ regardless

of the value of $|X_a|$, which might indicate a nonnegligible degree of the $K\bar{K}$ molecule for the $f_0(980)$ resonance.

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